Causal Confirmation Theory

Casual Structure and Evidential Impact in Probabilistic Reasoning

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With Can Konuk and Salvador Mascarenhas

Probabilistic Reasoning

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Theoretical perspectives on judgement under uncertainty:

- Heuristics and biases (Tversky and Kahneman, 1974)
- Confirmation theory and evidential impact (Crupi et al., 2008)
- Causal model theory (Krynski and Tenenbaum, 2007)

Reasoning

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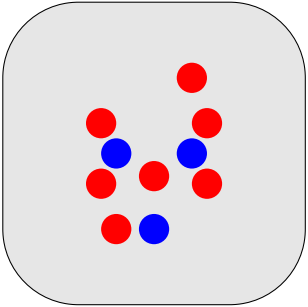

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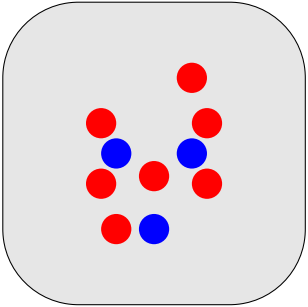

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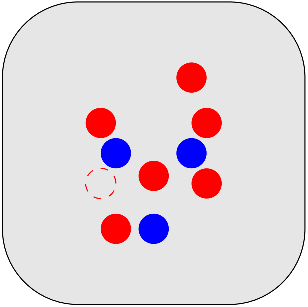

(b) Balls in a box and one is stuck.

Common confirmation measures[1]:

[1] Crupi and Tentori (2016)

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1. likelihood ratio: $\frac{p(e|h)}{p(e|\neg h)}$

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1. likelihood ratio: $\frac{p(e|h)}{p(e|\neg h)}$
2. probability ratio: $\frac{p(h|e)}{p(h)}$
3. evidential impact: $\frac{p(e|h)-p(e|\neg h)}{p(e|h)+p(e|\neg h)}$

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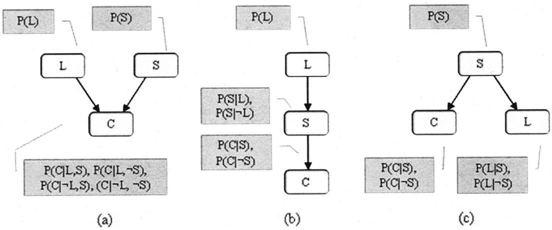

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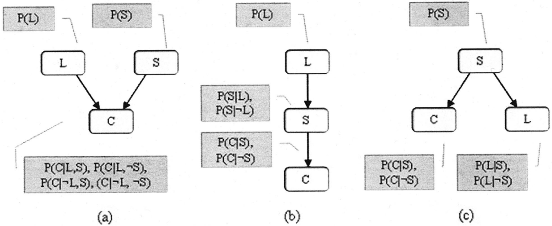

(a) $P(C|L) = \sum_S P(S)P(S|L,S)$

(b) $P(C|L) = \sum_S P(S|L)P(C|S)$

(c) $P(C|L) = \sum_S \frac{\sum_L P(S)P(C|S)P(L|S)}{\sum_C P(S)P(C|S)P(L|S)}$

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Can the two theories be pulled apart?
What are the interactions (if any) between these two theories?

Design

Statistical Proportions

Three ways of organizing base-rate and likelihood proportions:

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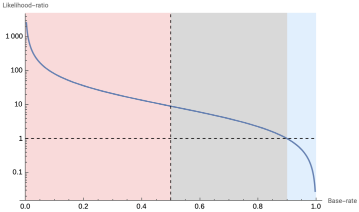

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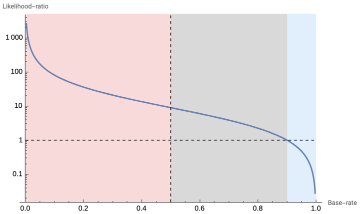

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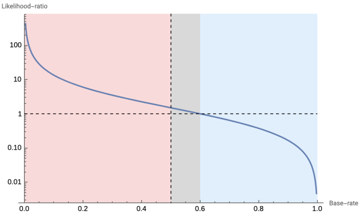

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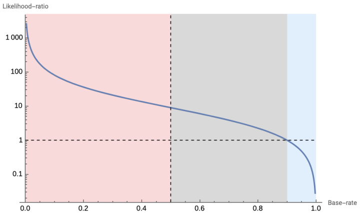

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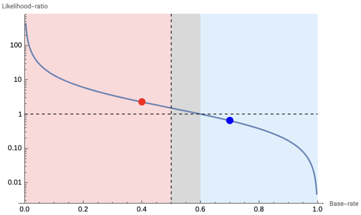

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Background Story

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An ancient Mesopotamian village engages in an annual parade to celebrate the harvest season.

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An ancient Mesopotamian village engages in an annual parade to celebrate the harvest season.
Several men are selected to take part in the parade either as **members** or as **leaders**.

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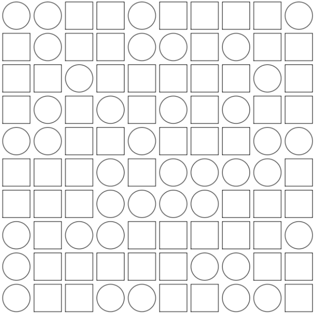
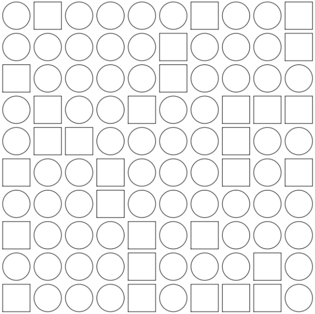


(a) base-rates: 40% Youths



(b) Base-rates: 70% Youths

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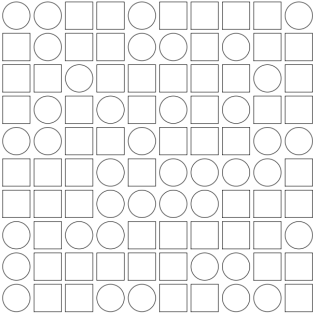

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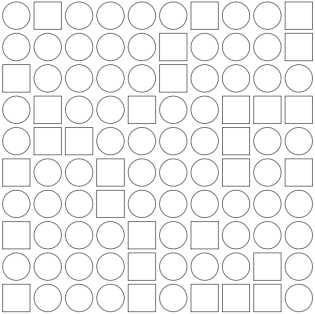

(b) Base-rates: 70% Youths

Youths are represented as circles and Elders as squares.

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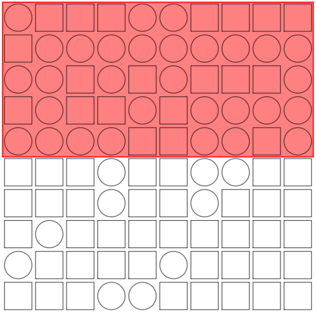
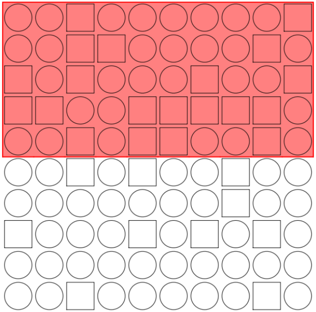
Selections and Causal Explanations



(a) Evidence: 75% Youths, 33% Elders



(b) Evidence: 43% Youths, 66% Elders

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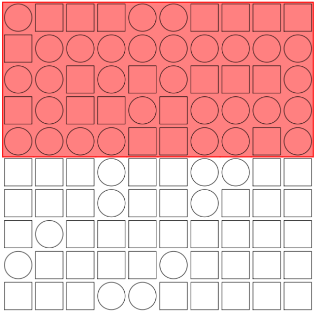

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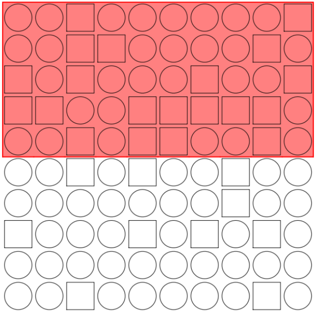


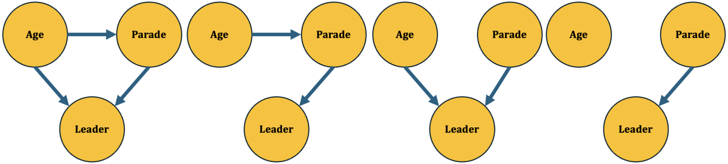
(b) Evidence: 43% Youths, 66% Elders

- Because of their greater wisdom the Elders fared better... $P(\textbf{leader}|\textbf{Elder}) > P(\textbf{leader}|\textbf{Youth})$.
- Because of their greater athleticism the Youths fared better... $P(\textbf{leader}|\textbf{Elder}) < P(\textbf{leader}|\textbf{Youth})$.

Mesopotamia

Selection proportion statistics

Proposed causal structures by condition:



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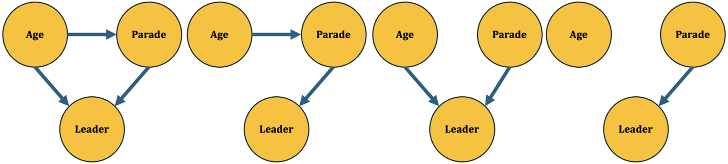


Condition	PRIORS-POST		LIK-POST	
Age class	Youths	Elders	Youths	Elders
Base-Rate	70	30	40	60
Likelihood	43%	66%	75%	33%
Posteriors	30/50 Youths (60%), 20/50 Elders (40%)			

Table: Base-rates and likelihoods for each condition.

Mesopotamia

Questions

Posterior:

(1) **Balthazar, a participant selected for the parade, got the red mask. Is Balthazar more likely to be an Elder or a Youth?**

Base-rates:

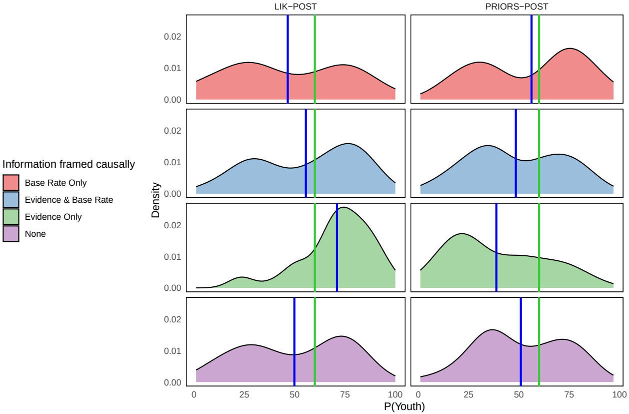
(2) Among the originally selected parade members (before the bull ritual), were there more Elders or Youths?

Likelihoods:

(3) During the selection of the leaders of the parade, which group fared better in the bull ritual, Youths or Elders?

Results

Posterior Accuracy



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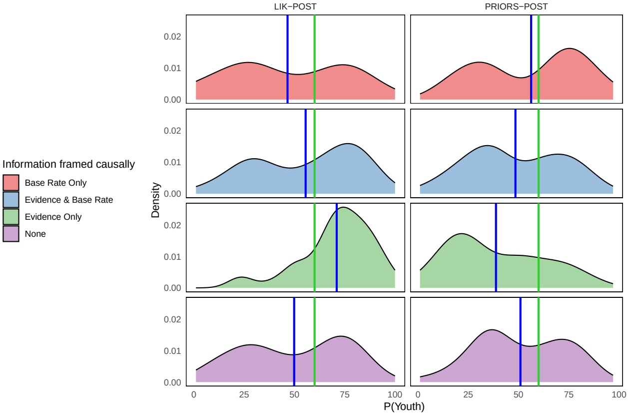

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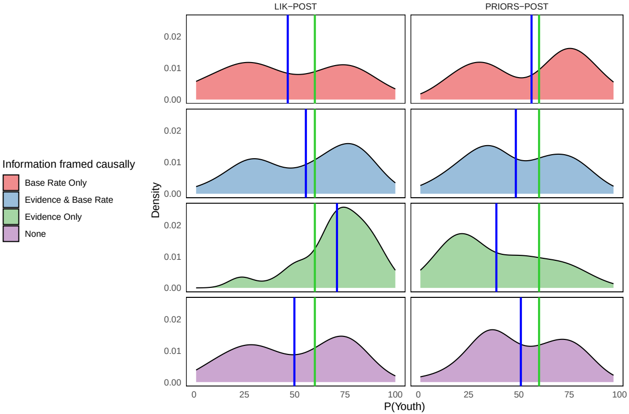

- Subjects give significantly higher judgements to $p(Youth|Leader)$ when the evidence is in line with the posterior
- Causality of the evidence matters more than that of the base-rate

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Probabilistic reasoning is causal insofar as the confirmation measures are computed as a function of the causal model that is represented.

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Thank you

Bibliography

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